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Mahgoub Decomposition Method for Solving Nonlinear Delay Differential Equations

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Abstract-This paper presents Mahgoub Decomposition Method for the solution of nonlinear Delay Differential Equations. The proposed method is a combination of Mahgoub Transform and Adomian Decomposition method. In this work, the solution is obtained as a series by first applying the Mahgoub Transform to the DDEs and then decomposing the nonlinear term by finding Adomian polynomials. Numerical examples are given to illustrate the effectiveness of our proposed method.
Index Terms-Mahgoub transform, Adomian polynomial, Nonlinear delay differential equations.

1. INTRODUCTION

Delay Differential Equations (DDEs) are a type of differential equations in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times. DDEs appear in chemical kinetics [1], population dynamics [2] and traffic models [3] and in several fields.
The general first order DDEs of the form

$$y'(t) = f(t, y, y(t-\tau)), \quad t > t_0 \quad (1)$$
$$y(t) = \emptyset(t), \quad t \le t_0$$

Here, $\emptyset(t)$ is the initial function, $\tau(t, y(t))$ is called the delay term. If the delay is a constant, then it is called constant delay. If it is function of time t, then it is called time dependent delay. If it is a function of time t and $y(t)$, then it is called the state dependent delay.
The general theory of DDEs have been widely developed by Bellman and Cooke [4], Hale [5], Driver [6]. Many numerical methods have been proposed by the researchers for the solution of DDEs. Some notable methods are Chebyshev Series Method [7], Variational Iteration Method [8], Runge-Kutta Method [9], Modified Power Series Method [10].
In this paper, Mahgoub Decomposition method is proposed to solve the nonlinear DDEs. This method is the combination of Mahgoub transform and Adomian decomposition method which is capable to solve nonlinear differential equations. This paper has been organized as follows: In Section 2, the Mahgoub transform and its fundamental properties have been discussed. In Section 3, Mahgoub Decomposition Method has been proposed for nonlinear DDEs. In Section 4, nonlinear numerical examples have been provided to demonstrate the efficiency of the proposed method.

2. PRELIMINARIES AND NOTATIONS

Mahgoub transform was recently introduced by Mohand Mahgoub [11] in 2016. This transform is derived from the classical Fourier integral. It is defined for function of exponential order in the set A defined by:

$$A = \{f(t): \exists M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}\}$$

For a given function in the set A, M is constant and finite number, k_1, k_2 either finite or infinite. The Mahgoub transform is denoted by the operator $M(\cdot)$ defined by the integral equation:

$$M[f(t)] = H(v) = v\int_0^\infty f(t)e^{-vt}dt, \quad (2)$$
$$t \ge 0, k_1 \le v \le k_2$$

In this transform the variable v is used to factor the variable t in the argument of the function f. This transform has connection with Fourier, Laplace, Elzaki transforms.
Mahgoub Transform of simple functions is given below:

i. $M[1] = 1$
ii. $M[t] = \frac{1}{v}$
iii. $M[t^n] = \frac{n!}{v^n}$ where n is the positive integer.
iv. $M[e^{at}] = \frac{v}{v-a}$
v. $M[\sin at] = \frac{av}{v^2+a^2}$
vi. $M[\cos at] = \frac{v^2}{v^2+a^2}$

Mahgoub Transform for derivatives are:

i. $M[f'(t)] = vH(v) - vf(0)$
ii. $M[f''(t)] = v^2H(v) - vf'(0) - v^2f(0)$
iii. $M\left[f^{(n)}(t)\right] = v^{(n)}H(v) - \sum_{k=0}^{n-1} v^{n-k}f^{(k)}(0)$

3. CONSTRUCTION OF MAGHOUB DECOMPOSITION METHOD FOR DDEs

The Adomian Decomposition Method (ADM) has been developed by Adomian [12]. MaghoubDecomposition Method (MDM), which is the combination of Maghoub Transform and ADM, has been proposed here to solve the nonlinear DDEs. The solution is obtained as a series. First, we apply the

Maghoub transform to DDEs, then decomposing the nonlinear term by finding Adomian polynomials.

Consider the nonlinear delay differential equation in the form:

$$Ly + N(y, y_\tau) + R(y, y_\tau) = g, \text{for } t > 0,$$
$$y = \emptyset(t), \text{for } t \in [-\tau, 0].$$

where $y_\tau(t) = y(t-\tau)$ is delay term, L is easily invertible, N is the nonlinear part, R is the remaining part and g is the source term.

The nonlinear term shall be decomposed into an infinite series of Adomian polynomials as follows:

$$N(y, y_\tau) = \sum_{n=0}^{\infty} A_n$$

where $A_n = A_n(y_0, y_1, \dots\dots\dots, y_n)$ are called Adomian polynomials and A_n is classically suggested to the computed form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i y_i\,, \sum_{i=0}^{\infty}\lambda^i y_{\tau i}\right)\right]_{\lambda>0}$$

The Maghoub Decomposition algorithm is implemented for the solution of the following first order nonlinear initial value problem

$$y' = f(t, y, y(t-\tau)), \quad y(0) = \alpha$$

Apply the Maghoub transform on both sides of (3) and using initial condition we get,

$$M[y'] = M[f(y, y(t-\tau))]$$
$$M[y(t)] = \alpha + \frac{1}{v}M[f(y, y(t-\tau))]$$

The Maghoub decomposition technique now gives the solution as an infinite series,

$$M[\textstyle\sum_{n=0}^{\infty} y_n] = \alpha + \frac{1}{v}M[\sum_{n=0}^{\infty} A_n] \qquad (4)$$

where A_n is Adomian polynomials for nonlinear terms.

From Eqn. (4), we get the following recursive algorithm

$$M[y_0] = \alpha$$
$$M[y_{n+1}] = \tfrac{1}{v}M[A_n],\ n > 0.$$

By taking inverse Maghoub transform, we get $y_0, y_1, y_2, \dots\dots$ The analytical solution of nonlinear DDEs by MDM is given as an infinite series

$$y(t) = \textstyle\sum_{n=0}^{\infty} y_n(t).$$

By finding sufficient number of $y_n{}'s$ we get the numerical solution with good accuracy.

4. NUMERICAL EXAMPLES

Example 4.1:

Consider the first order nonlinear DDE

$$y'(t) = 1 - 2y^2\left(\frac{t}{2}\right), \quad y(0) = 0$$

The exact solution is $y(t) = \sin(t)$.

Applying Maghoub Transform to the above first order nonlinear DDE, we obtain

$$M[y(t)] = \frac{1}{v} - \frac{2}{v}M\left[y^2\left(\frac{t}{2}\right)\right]$$

Then using Adomian Decomposition Method, we obtain

$$M[\textstyle\sum_{n=0}^{\infty} y_n(t)] = \frac{1}{v} - \frac{2}{v}M[\sum_{n=0}^{\infty} A_n] \qquad (5)$$

From Eqn. (5), we get,

$$y_0(t) = t$$
$$y_1(t) = -\tfrac{t^3}{3!},$$
$$y_2(t) = \tfrac{t^5}{5!},$$
$$y_3(t) = -\frac{t^7}{7!},$$
$$\dots\dots$$

The infinite series solution becomes,

$$y = y_0 + y_1 + y_2 + y_3 + \cdots$$
$$y = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$$

which converges to the exact solution $y(t) = \sin(t)$ as $n \to \infty$.

Example 4.2:

Consider the third order nonlinear DDE

$$y'''(t) = -1 + 2y^2\left(\frac{t}{2}\right),$$
$$y(0) = 0, y'(0) = 1, y''(0) = 0 \qquad (3)$$

The exact solution is $y(t) = \sin(t)$.

Applying Maghoub Transform to the above third order nonlinear DDE, we obtain

$$M[y(t)] = \frac{1}{v} - \frac{1}{v^3} + \frac{2}{v^3}M\left[y^2\left(\frac{t}{2}\right)\right]$$

Then using Adomian Decomposition Method, we obtain

$$M[\textstyle\sum_{n=0}^{\infty} y_n(t)] = \frac{1}{v} - \frac{1}{v^3} + \frac{2}{v^3}M[\sum_{n=0}^{\infty} A_n] \qquad (6)$$

From Eqn. (6), we get

$$y_0(t) = t - \frac{t^3}{3!}$$
$$y_1(t) = \tfrac{t^5}{5!} - \tfrac{t^7}{7!} + \tfrac{5}{8}\tfrac{t^9}{9!},$$
$$y_2(t) = \frac{3}{8}\frac{t^9}{9!} - \frac{63}{2^6}\frac{t^{11}}{11!} + \frac{505}{2^{10}}\frac{t^{13}}{13!} - \frac{275}{2^{11}}\frac{t^{15}}{15!},$$
$$\dots\dots$$

The infinite series solution becomes,

$$y = y_0 + y_1 + y_2 + y_3 + \cdots$$
$$y = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots$$

which converges to the exact solution $y(t) = \sin(t)$ as $n \to \infty$.

5. CONCLUSION

In this paper we proposed Maghoub Decomposition Method to solve nonlinear delay differential equations. It is the combination of Maghoub transform and Adomian decomposition method to produce exact/approximate solutions of the nonlinear DDEs. Two examples have been considered to demonstrate the applicability of the proposed method. This method is very efficient to solve nonlinear DDEs and gives results with good accuracy.

(A.1)

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